Entanglement and secret-key-agreement capacities of bipartite quantum interactions and read-only memory devices

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Bipartite quantum interactions

Bipartite unitary interactions are the most elementary many-body interactions. Due to unavoidable interaction with environment, study of bipartite noisy interactions is pertinent.

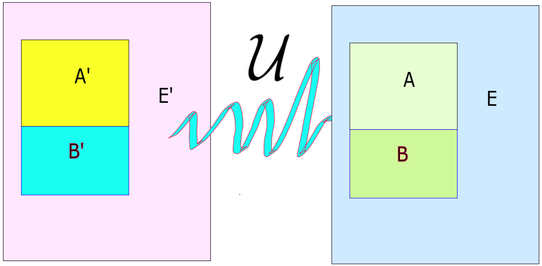

Figure: Systems of interest A' and B' interacting in presence of the bath E'.

- $\mathcal{U}$ is unitary transformation corresponding to underlying *interaction Hamiltonian* $\hat{H}$ among A', B', E'.
- Before action of interaction Hamiltonian $\hat{H}$: $\omega_{A'B'} \otimes \tau_{E'}$, where bath E' is in some fixed state and uncorrelated to $A'B'$.
- After action of $\hat{H}$:

$$\rho_{ABE} := \mathcal{U}_{A'B'E' \to ABE}(\omega_{A'B'} \otimes \tau_{E'}).$$

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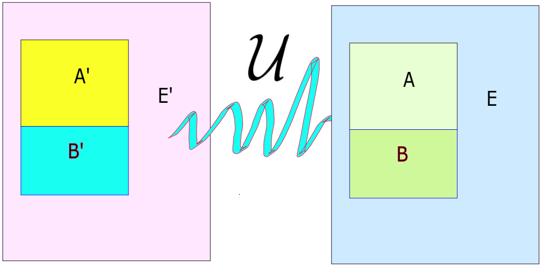

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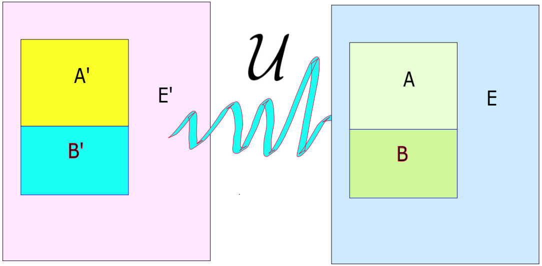

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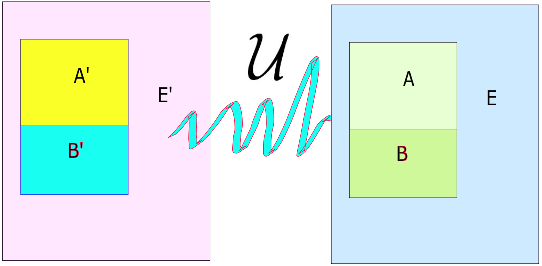

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A bipartite quantum channel $\mathcal{N}_{A'B'\to AB}$ is a completely positive, trace-preserving map that transforms composite system $A'B'$ to AB.

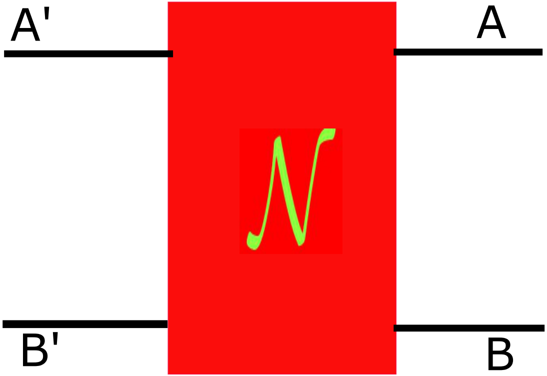

Figure: Two parties of interest: Alice holds A', A and Bob holds B', B.

- When A', A are held by Alice and B', B are held by Bob, bipartite channel $\mathcal{N}$ is called bidirectional channel.
- It corresponds to noisy bipartite interaction, when bath is inaccessible.
- For all input state $\omega_{A'B'}$:

$$\mathcal{N}(\omega_{A'B'}) = \rho_{AB}, \text{ where}$$

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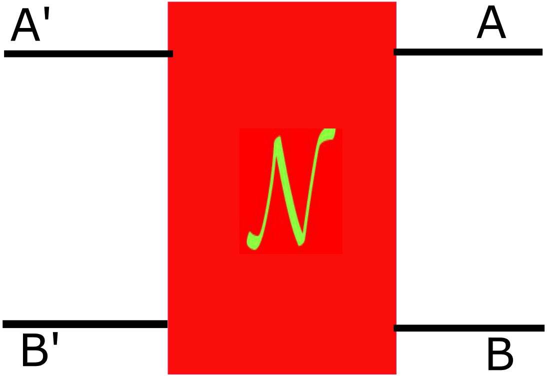

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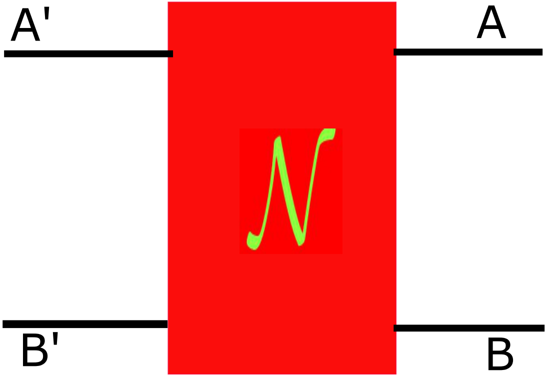

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Motivation

Bidirectional channels:

- Simple model of quantum network with 2 clients, Alice and Bob.
- Model for quantum gates – CNOT, SWAP, etc.– in noisy intermediate-scale quantum (NISQ) computers.

Entanglement may increase, decrease or not change due to bipartite quantum interactions.

Entanglement distillation: Maximally entangled states are useful resource for several information processing tasks: quantum key distribution, quantum teleportation, etc.

Secret key distillation: Need for secure communication protocols between two parties over network – private reading.

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Goal

- Two different information-processing tasks relevant for bipartite quantum interactions:
    1. Entanglement distillation: generation of singlet state from two separated systems.
    2. Secret key agreement: generation of maximal classical correlation between two separated systems, such that there's no correlation with the bath.
- New secure communication protocol between two parties, called private reading.

Non-asymptotic capacity of a channel $\mathcal{N}$ for a task: Maximum rate at which a given task can be accomplished by allowing the use of $\mathcal{N}$ a finite number of times.

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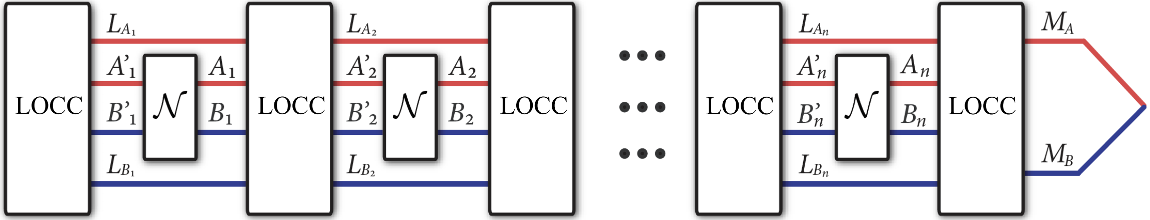
Secret key generation over bidirectional channel



- LOCC-assisted bidirectional secret-key-agreement capacity $[\mathcal{P}^{2\to 2}_{LOCC}(\mathcal{N}_{A'B'\to AB}).]$
- Bidirectional max-relative entropy of entanglement:

$$E^{2\to 2}_{\max}(\mathcal{N}_{A'B'\to AB}) = \sup_{\psi_{S_AA'}\otimes\varphi_{B'S_B}} E_{\max}(S_AA;BS_B)_{\mathcal{N}(\psi\otimes\varphi)},$$

where $\psi_{S_AA'}, \varphi_{B'S_B}$ are pure states, $S_A \simeq A', S_B \simeq B'$,
$E_{\max}(A:B)_\rho = \min_{\sigma_{AB}\in \mathrm{SEP}} D_{\max}(\rho\|\sigma)$, such that $D_{\max}(\rho\|\sigma) = \inf\{\lambda : \rho \le 2^\lambda\sigma\}$.

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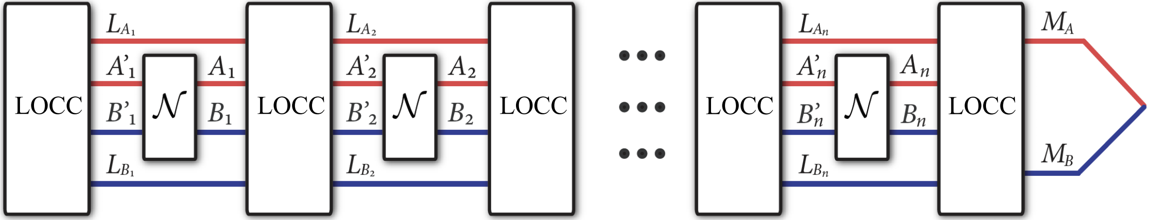

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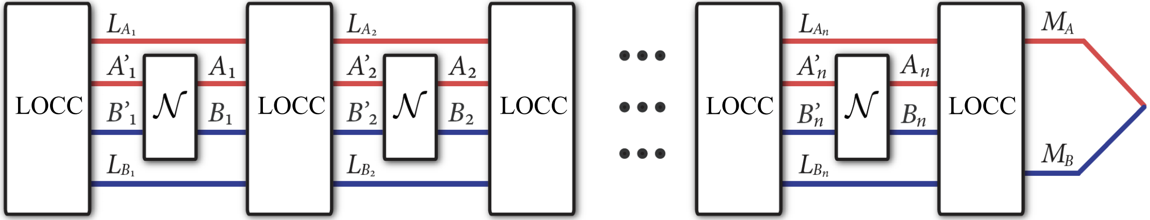

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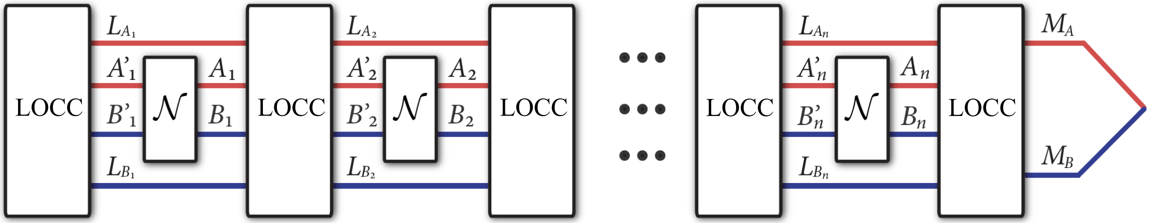

Theorem (LOCC-assisted secret key agreement)

$$\frac{1}{n}\log_2 M \leq E_{\max}^{2\to 2}(\mathcal{N}) + \frac{1}{n}\log_2\left(\frac{1}{1-\varepsilon}\right).$$

- $P_{LOCC}^{2\to 2}(\mathcal{N}_{A'B'\to AB}) \leq E_{\max}^{2\to 2}(\mathcal{N}_{A'B'\to AB})$, and this upper bound is in fact a strong converse bound.

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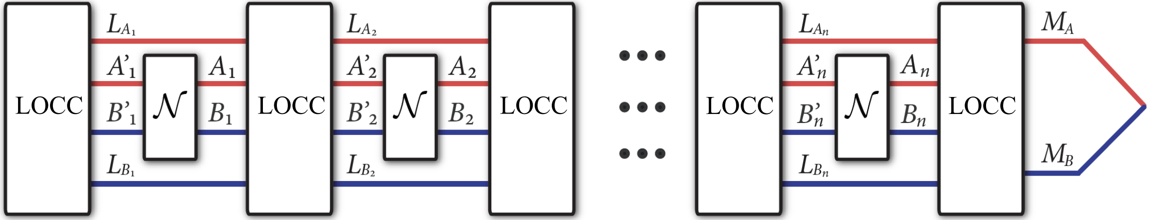

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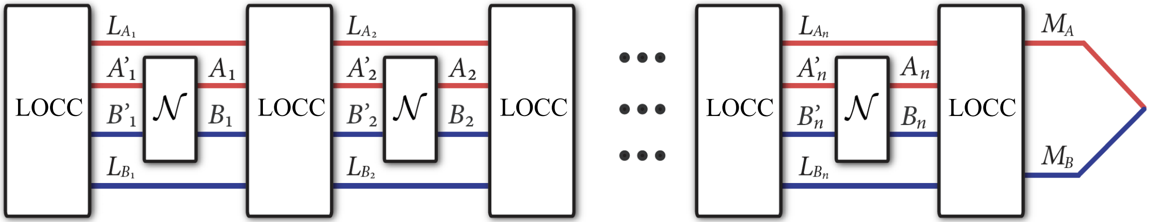

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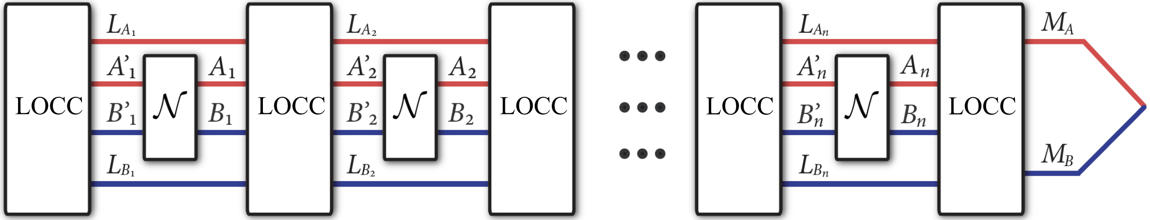

- Private states [HHHO05,HHHO09]: State $\gamma_{S_A K_A : K_B S_B}$ containing $\log_2 K$ private bits.
- Success probability in privacy test: $\text{Tr}\{\Pi^\gamma \omega\} \geq 1 - \varepsilon$. By [HHHO05,HHHO09], $\text{Tr}\{\Pi^\gamma \sigma\} \leq \frac{1}{K}$ for $\sigma \in \text{SEP}$.
- Main observation: $E_{\max}^{2\to 2}$ is not enhanced by amortization.

$$E_{\max}(S_A A; B S_B)_\sigma \leq E_{\max}(S_A A'; B' S_B)_\rho + E_{\max}^{2\to 2}(\mathcal{N}_{A'B'\to AB}),$$

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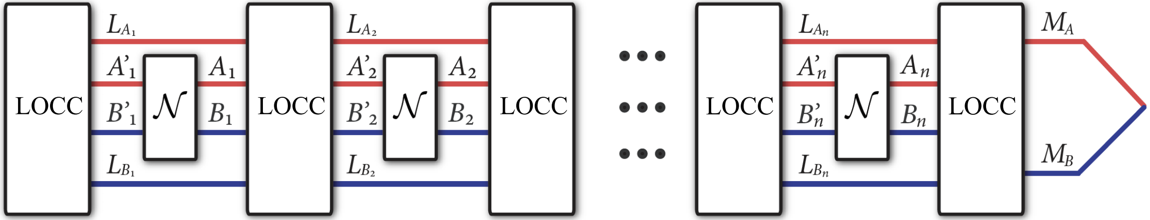

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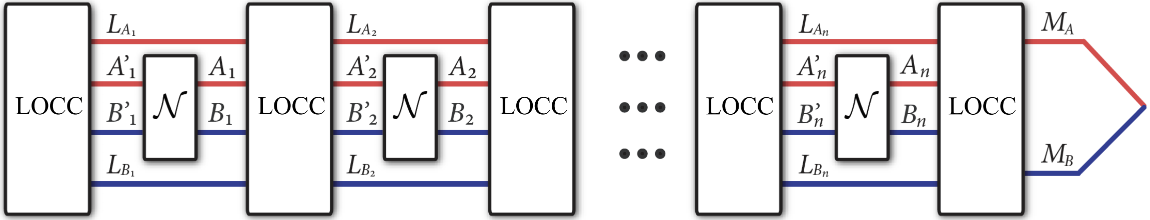

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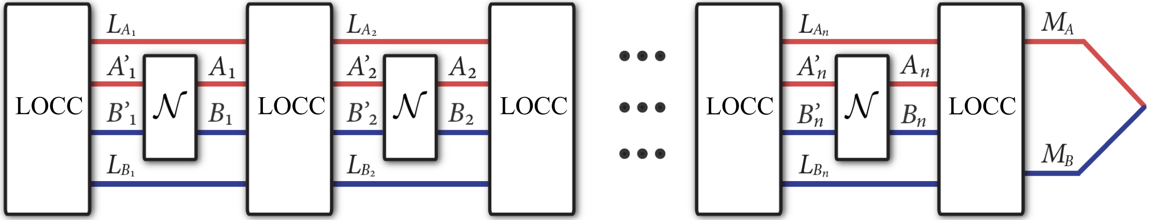

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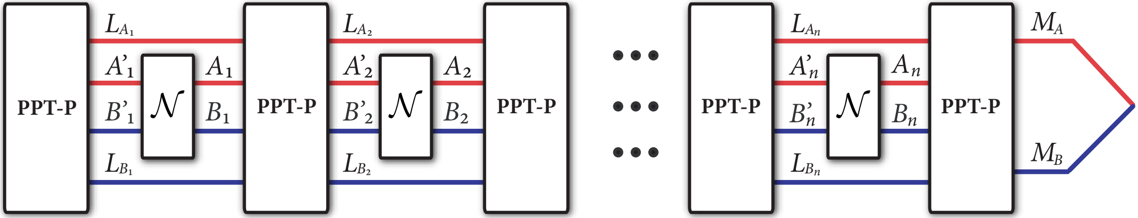

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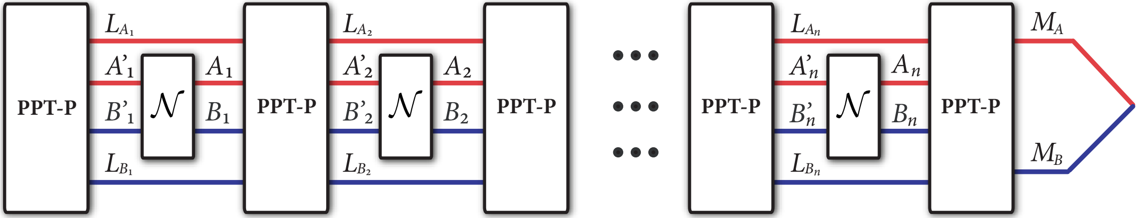

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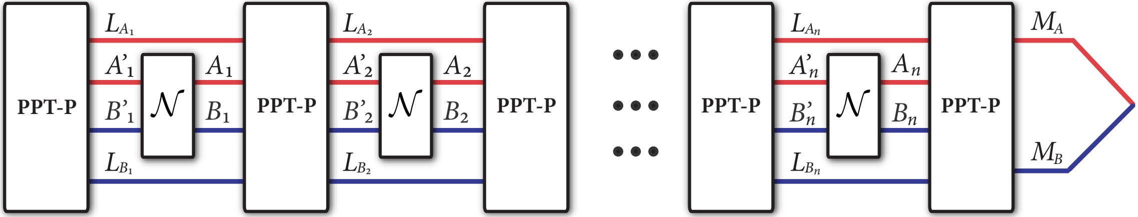

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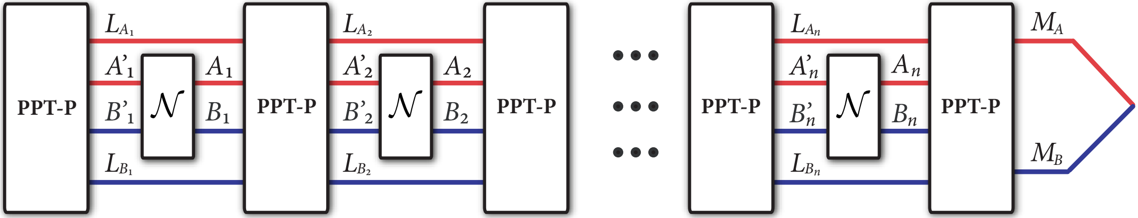

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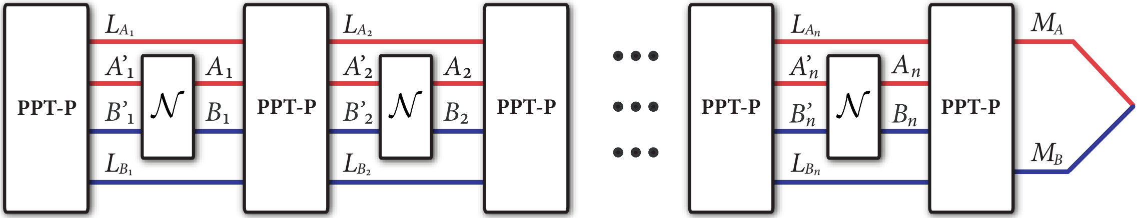

Theorem (PPT-assisted distillable entanglement generation)

$$\frac{1}{n}\log_2 M \leq R_{\max}^{2\to 2}(\mathcal{N}) + \frac{1}{n}\log_2\left(\frac{1}{1-\varepsilon}\right).$$

- $Q_{PPT}^{2\to 2}(\mathcal{N}_{A'B'\to AB}) \leq R_{\max}^{2\to 2}(\mathcal{N}_{A'B'\to AB})$, and this upper bound is in fact a strong converse bound.

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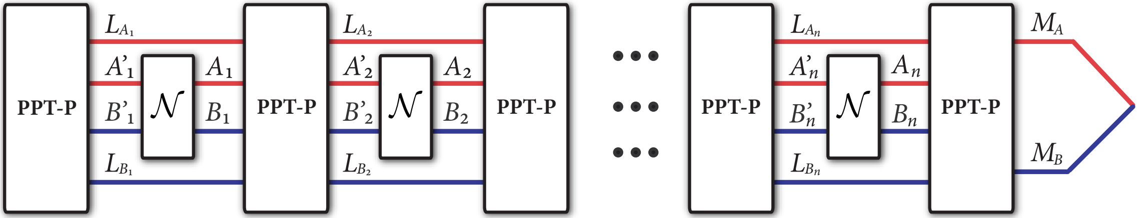

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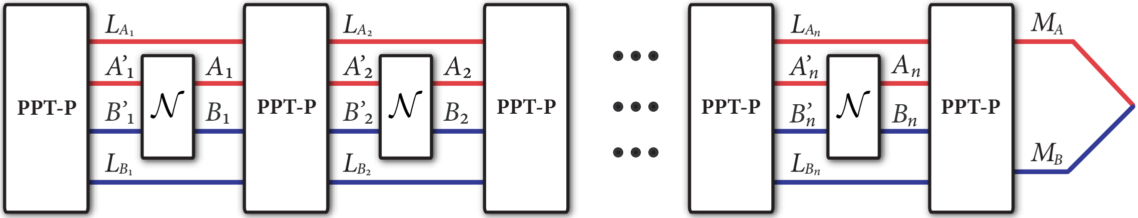

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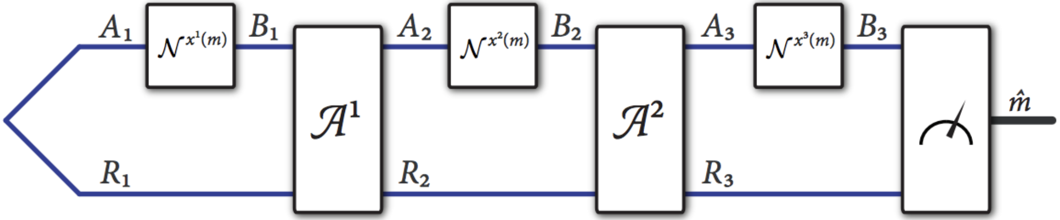
- First recall: (Quantum) Reading [BRV00,Pir11] of memory devices.
- Memory device: message encoded into a sequence of channels from a memory cell $\mathcal{S}_{\mathcal{X}} = \{\mathcal{N}^{x}_{B'\to B}\}_{x\in\mathcal{X}}$.
- Alice encodes $m \in \mathcal{M}$ into codewords $(x_1(m), ..., x_n(m))$, and sets the device to $\left(\mathcal{N}^{x_1(m)}_{B'\to B}, ..., \mathcal{N}^{x_n(m)}_{B'\to B}\right)$.
- Bob can enter quantum states and do channel discrimination to learn m. Natural to employ adaptive strategy [DW17].



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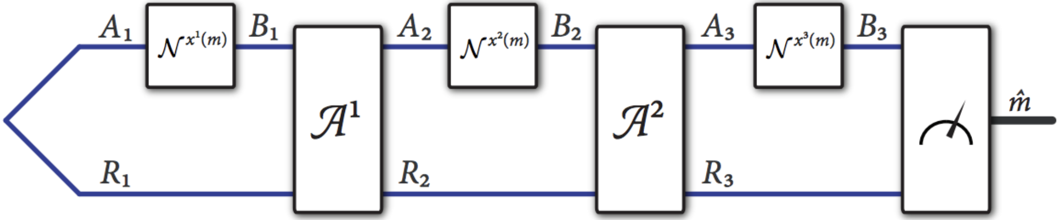

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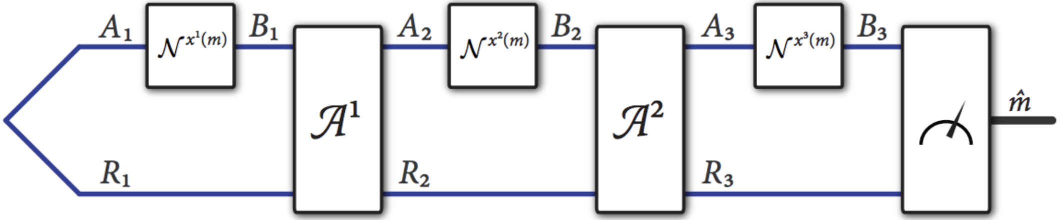

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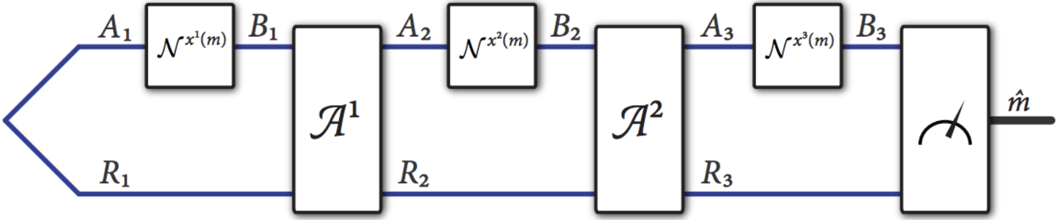

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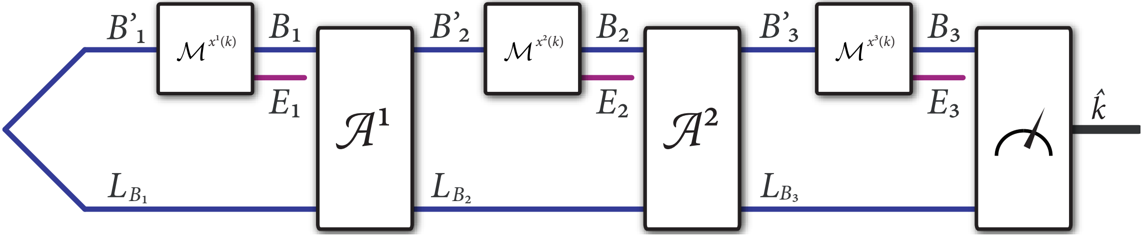

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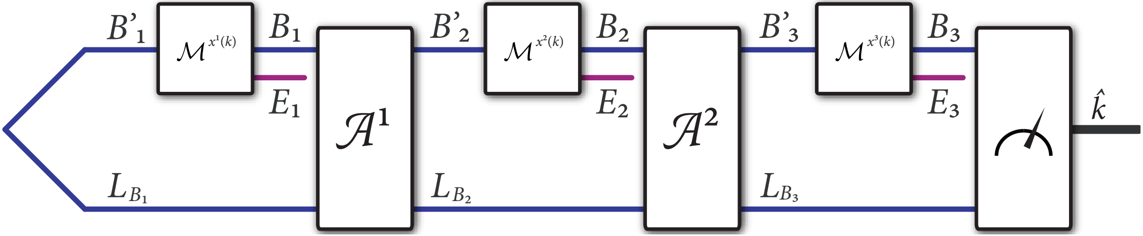

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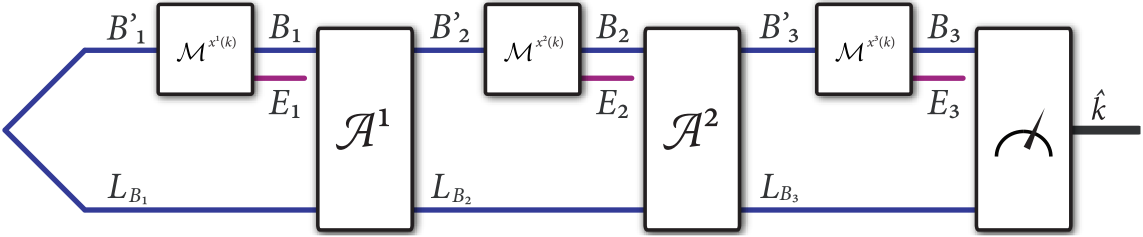

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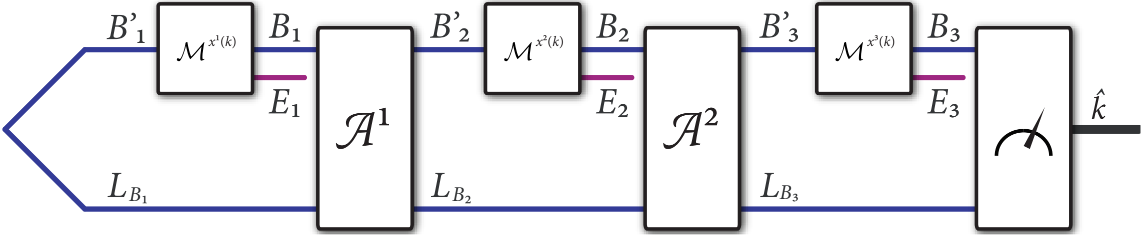

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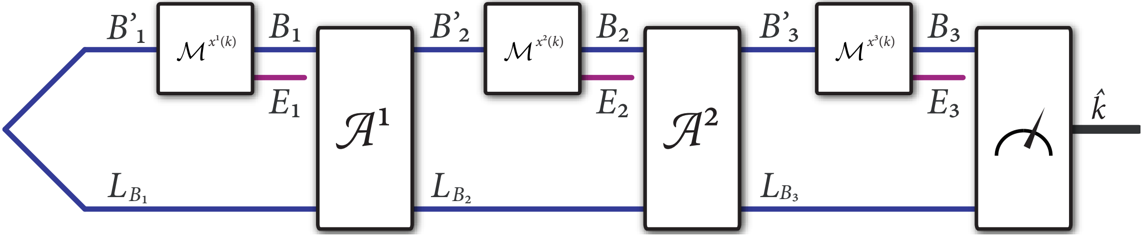

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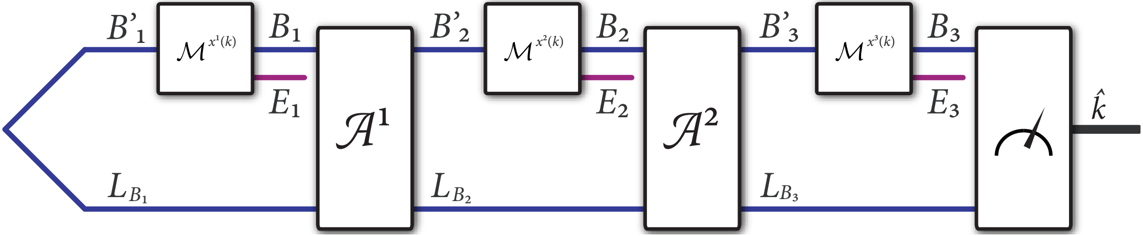

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Private reading capacity

The strong converse private reading capacity $\widetilde{P}^{\text{read}}(\bar{\mathcal{S}}^{\text{iso}}_{\mathcal{X}})$ of an isometric wiretap memory cell $\bar{\mathcal{S}}^{\text{iso}}_{\mathcal{X}} = \{\mathcal{U}^{\mathcal{M}^x}_{B'\to BE}\}_{x\in\mathcal{X}}$ is bounded from above as

$$\widetilde{P}^{\text{read}}(\bar{\mathcal{S}}^{\text{iso}}_{\mathcal{X}}) \leq E^{2\to 2}_{\max}(\mathcal{N}^{\bar{\mathcal{S}}}_{XB'\to XB}),$$

where

$$\mathcal{N}^{\bar{\mathcal{S}}}_{XB'\to XB}(\cdot) := \operatorname{Tr}_E\left\{U^{\bar{\mathcal{S}}}_{XB'\to XBE}(\cdot)\left(U^{\bar{\mathcal{S}}}_{XB'\to XBE}\right)^{\dagger}\right\},$$

such that

$$U^{\bar{\mathcal{S}}}_{XB'\to XBE} := \sum_{x\in\mathcal{X}} |x\rangle\langle x|_X \otimes U^{\mathcal{M}^x}_{B'\to BE}.$$

Conclusion

- We derived upper bounds on entanglement generation and secret-key-agreement capacities over bidirectional channels. Sizes of reference systems are same as size of input systems (Open question in [BHLS03]).
- Obtain tighter upper bounds for channels obeying certain symmetries, see [DBW17].
- Introduced secure protocol for reading of memory devices under scrutiny of an eavesdropper. Both upper and lower bounds for this protocol can be found in [DBW17].

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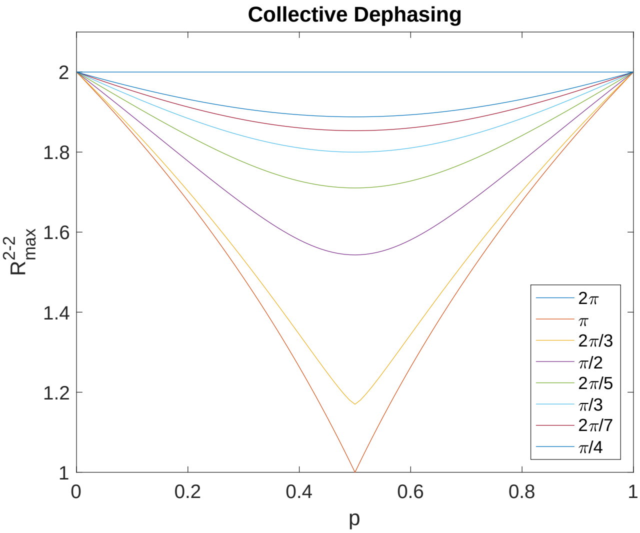
SWAP and Collective dephasing



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- Swap operator $S = \sum_{ij} |ij\rangle\langle ji|$ and collective dephasing:

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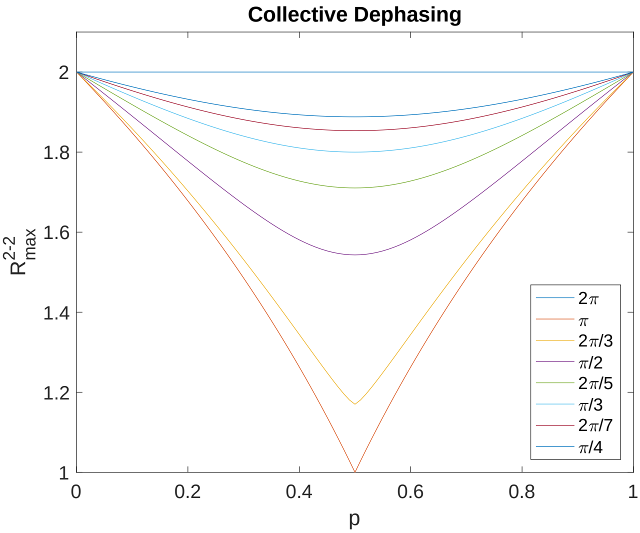

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